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Amplitude-squared squeezing and photon statistics in second and third harmonic generations

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Abstract Squeezing and Photon antibunching of the field amplitude in harmonic generation has been investigated by Mandel [1] Later, squeezing of the square of the field amplitude of the fundamental mode (obeying sub-poissonian photon statistics) during second harmonic generation (SHG) was studied by Hillery[2] The present study is aimed at obtaining results for squeezing of the square of the field amplitude and photon statistics in the second and third harmonics It has been found that in the process of SHG, there is squeezing of the square of the field amplitude of the second harmonic mode when calculated upto $(gt)^4$ It depends on the values of the interaction time 't' and the coupling parameter 'g' and is independent of the phase of the fundamental mode Also, the statistics of the second harmonic is poissonian However, there is no squeezing effect in the third harmonic upto $(\lambda t)^3$

Keywords Squeezed states, harmonic generation, sub-poissonian statistics

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1. Introduction

Since 1980's, lot of work has appeared in literature on the generation of squeezed states of the electromagnetic field and on their experimental detection [1-8]. Such states have less noise than a coherent state in one of the field quadratures. So these find applications in optical communication systems [9-10] and gravitational radiation detector [11-13].

Mandel [1] considered the case in which a beam of light propagates through a non-linear crystal and found that during the process of second harmonic generation the field amplitude of the fundamental mode becomes squeezed. However, he found no squeezing in the second harmonic mode. Hillery [2], introduced the higher order (higher – power field amplitude) squeezing and showed that the normal squeezing of second harmonic depends directly upon the amplitude-squared squeezing of the incident light in the fundamental mode. He has also

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shown that amplitude-squared squeezed states can be of use in reducing noise in the output of certain non-linear optical devices [14]. Hong and Mandel [15-16], however, have obtained the results for normal squeezing corresponding to N-th power of the field amplitude.

Earlier [2] it was shown that in the process of second harmonic generation, there is squeezing of the square of the field amplitude of the fundamental mode for certain phase values only. The present paper, on the other hand, shows squeezing in the second harmonic mode which is dependent on suitable values of the interaction time 't' and coupling parameter 'g' between the modes without obeying sub-poissonian photon statistics when calculated upto to $(gt)^4$. However, no squeezing effect is found in the third harmonic upto $(\lambda t)^2$.

2. Definition of squeezing of the square of field amplitude

Consider a single mode of electromagnetic field of frequency ω with creation and annihilation operators a^+, a, respectively. Amplitude squared squeezing is defined in terms of the operators

$$X_1 = \tfrac{1}{2}(A^2 + A^{+2}), \tag{1}$$

$$X_2 = \tfrac{1}{2}i(A^2 - A^{+2}),$$

where A, A^+ are slowly varying operators defined by

$A = a\exp(i\omega t)$ and $A^+ = a^+\exp(-i\omega t)$, obeying the same commutation rules as a, a^+ [2].

These operators obey the commutation relation

$$[X_1, X_2] = i(2n+1) \tag{2}$$

which leads to uncertainty relation

$$\Delta X_1 \ \ \Delta X_2 \geq \langle n + \tfrac{1}{2} \rangle, \tag{3}$$

where $n = A^+A$ is the usual number operator and $\Delta X = (\langle (\Delta X)^2 \rangle)^{1/2}$.

Amplitude squared squeezing is said to exist in the X_i variable if

$$(\Delta X_i)^2 < \langle n + \tfrac{1}{2} \rangle \quad \text{for } i = 1 \text{ or } 2\,. \tag{4}$$

Systems with Squeezed states, as defined by eq. (4), exhibit non-classical features. It can be shown by expressing $(\Delta X_1)^2$ in terms of the P-representation (Glauber-Sudershan representation) of the state. This may be seen by the following explicit calculation

$$(\Delta X_1)^2 = \tfrac{1}{4} \langle (A^2 + A^{+2})^2 - \langle A^2 + A^{+2} \rangle^2 \rangle$$

$$= \tfrac{1}{4}[\langle (A^4 + A^{+4} + 2A^{+2}A^2 + 4A^+A + 2 - \langle A^2 + A^{+2} \rangle^2]$$

$$(\Delta X_1)^2 = \langle n + \tfrac{1}{2} \rangle + \tfrac{1}{4}\int d^2\alpha\, P(\alpha) \times [\exp(-2i\omega t)\,\alpha^{*2} + \exp(2i\omega t)\alpha^2$$

$$- \langle A^2 + A^{+2} \rangle]^2 \tag{5}$$

where $d^2\alpha = d(\mathrm{Re}\,\alpha)\, d(\mathrm{Im}\,\alpha)$ and $P(\alpha)$ is the coherent-state quasi probability function. A classical state, whose P-representation is non-negative definite, must always satisfy the relation

$(\Delta X_1)^2 \geq \langle n + ½\rangle$. Therefore, the condition for squeezing $(\Delta X_1)^2 < \langle n + ½\rangle$ requires that $P(\alpha)$ be a non-positive definite function. A coherent state is defined as that for which

$$(\Delta X_1)^2 = (\Delta X_2)^2 = \langle n + ½\rangle .$$

3. Squeezing in second harmonic generation

Second harmonic generation is a process in which an incident beam of fundamental frequency ω interacts with a non-linear medium to produce a harmonic at 2ω. The effective interaction Hamiltonian in the process of second harmonic generation in a non-dissipative medium has the form $[\hbar = 1]$,

$$H = \omega n_1 + 2\omega n_2 + g\,[a_1^{+2} a_2 + a_2^{+} a_1^{2}\,], \tag{6}$$

where g is the mode coupling constant which involves the characteristics of the non-linear medium (g depends on the second-order susceptibility). The suffixes 1,2 refer to the fundamental mode of frequency ω and to the harmonic mode of frequency 2ω respectively. The operators a_1, a_2, a_1^+, a_2^+, and n_1, n_2 are annihilation, creation and number operators for the two modes respectively.

The slowly varying operators for the two modes are defined by

$$A = a_1 \exp(i\omega t), \qquad B = a_2 \exp(2i\omega t) \tag{7}$$

Eq. (6) leads to the coupled Heisenberg eqs. of motion [2],

$$dA/dt = -2ig\,A^{+}B, \quad dB/dt = -ig\,A^{2}. \tag{8}$$

Since the effective interaction time between modes which is of the order of the propagation time of the light through the non-linear crystal, is sufficiently short, one can make use of Taylor's expansion to solve eqn. (8). Thus, we obtain

$$A(t) = A(0) - 2igt A^{+}(0)B(0) + 2g^2t^2\,[\,n_2(0)A(0) - ½ n_1(0)A(0)\,] + \ldots \tag{9}$$

and

$$B(t) = B(0) - igtA^{2}(0) - 2g^2t^2\,[\,n_1(0)B(0) + ½B(0)\,] + \ldots\ldots, \tag{10}$$

where $n_1 = A^{+}A$ and $n_2 = B^{+}B$

During the process of second harmonic generation, we assume the initial quantum state of the system as a product of the coherent state $|\alpha_1\rangle$ for the fundamental mode 1 and the vacuum state for the harmonic mode 2 *i.e.* $|\alpha_1\rangle_1 |0\rangle_2$, we have

$$A|\alpha_1\rangle_1 = \alpha_1|\alpha_1\rangle_1 \text{ and } B|0\rangle_2 = 0. \tag{11}$$

With the use of eqs. [(9)–(11)], it has been shown that [2, 14]

$$[\,\Delta X_{1A}(t)\,]^2 - \langle n_1(t) + ½\rangle = -2g^2t^2\,|\alpha_1|^4 \cos 4\theta + \mathrm{oh}(gt)^3, \tag{12}$$

$$\langle(\Delta n_1(t))^2\rangle - \langle n_1(t)\rangle = -2g^2t^2\,|\alpha_1|^4 + \mathrm{oh}(gt)^3, \tag{13}$$

$$\langle(\Delta n_2(t))^2\rangle - \langle n_2(t)\rangle = \mathrm{oh}(gt)^6, \tag{14}$$

where $\alpha_1 = |\alpha_1|e^{i\theta}$ and X_{1A} is the real quadrature component of the square of the complex field amplitude of the fundamental mode.

Clearly from eqs. (12), (13) and (14) squeezing in this case is for $\cos 4\theta$ positive with sub-poissonian photon statistics. However, the statistics in the second harmonic is essentially indistinguishable from poissonian.

In order to define the amplitude squared squeezing affect in second harmonic, we define the operators

$$Y_{1B}(t) = \tfrac{1}{2}\,[\,B^2(t) + B^{+2}(t)],$$
$$Y_{2B}(t) = \tfrac{1}{2}\,i\,[\,B^2(t) - B^{+2}(t)], \tag{15}$$

which correspond to real and imaginary parts of the square of the complex field amplitude of the second harmonic mode. With the use of eqs. (10) and (15), we obtain upto $(gt)^2$ as

$$< Y_{1B}(t) > = -\tfrac{1}{2}\,g^2t^2\,(\alpha_1{}^4 + \alpha_1{}^{*4}) + \mathrm{oh}(gt)^3 \tag{16}$$

and

$$< (\,Y_{1B}(t) >)^2 = \tfrac{1}{2} + g^2t^2\,|\alpha_1|^4 + \mathrm{oh}(gt)^3. \tag{17}$$

Therefore,

$$[\,(\Delta\,Y_{1B}(t))\,]^2 = \tfrac{1}{2} + g^2t^2\,|\alpha_1|^4 + \mathrm{oh}(gt)^3. \tag{18}$$

Since

$$< n_2(t) > = g^2t^2\,|\alpha_1|^4 + \mathrm{oh}(gt)^3. \tag{19}$$

Thus

$$[\,(\Delta\,Y_{1B}(t))\,]^2 = < n_2\,(t) + \tfrac{1}{2} > + \mathrm{oh}(gt)^3. \tag{20}$$

Initially at time $t = 0$, the harmonic mode was assumed to be in the vacuum state, as the field propagates through the non-linear crystal, no. of photons of 2ω increases and the field at an instant of time t becomes coherent. However, if one accounts the effective time of interaction (t) and the coupling parameter (g) between different modes, one has upto order $(gt)^4$,

$$[\,(\Delta\,Y_{1B}(t)\,]^2 - < n_2\,(t) + \tfrac{1}{2} > = -2/3\;g^4t^4\,(2\,|\alpha_1|^6 + |\alpha_1|^4\,) \tag{21}$$

which shows the squeezing of the square of field amplitude in the second harmonic.

4. Third harmonic generation

Third harmonic generation is a process in which a fundamental beam of frequency ω interacts a non-linear medium to produce a harmonic at 3ω. The effective interaction Hamiltonian in this process has the form [$\hbar = 1$]

$$H = \omega\,n_1 + 3\omega\,n_3 + \lambda\,(a_1{}^{+3}a_3 + a_3{}^{+3}a_1)\,, \tag{22}$$

where λ stands for coupling constant and a_3, $a_3{}^+$ are annihilation and creation operators for the mode at frequency 3ω with $C = a_3$ expt $(3i\,\omega t)$ as slowly varying operator.

Eq. (22) leads to coupled Heisenberg eqs. of motion [17],

$$dA/dt = -\,3i\lambda\,A^{+2}\,C,$$
$$dC/dt = -\,i\lambda\,A^3\,. \tag{23}$$

With the assumption of short interaction time between modes, by making use of Taylor's expansion,

$$A(t) = A(0) - 3i\lambda A^{+2}(0)C(0) + 3/2\,\lambda^2 t^2 (6n_1(0)A(0)n_3(0) + 6A(0)n_3(0) - 2n_1^2(0)A(0) + n_1(0)A(0)) + \ldots\ldots\ldots, \tag{24}$$

$$C(t) = C(0) - i\lambda t A^3(0) - 3/2\,\lambda^2 t^2 (3n_1^2(0) + 3n_1(0) + 2)A(0) + \ldots\ldots\ldots \tag{25}$$

Again, we define the quadrature components of the field at 3ω as

$$Z_{1c}(t) = \tfrac{1}{2}\,[\,C^2(t) + C^{+2}(t)],$$

$$Z_{2c}(t) = \tfrac{1}{2}i\,[\,C^2(t) - C^{+2}(t)], \tag{26}$$

which correspond to the real and imaginary parts, respectively, of the square of the field amplitude of third harmonic.

With the use of eqs. (25) and (26), and by assuming the coherent state for the fundamental mode with amplitude α_2 and the vacuum state for the harmonic mode 3 during the third harmonic generation *i.e.* $A\,|\,\alpha_2>_1 = \alpha_2\,|\,\alpha_2>_1$ and $C|0>_3 = 0$, we obtain

$$< Z_{1c}(t) > = -\tfrac{1}{2}\lambda^2 t^2 (\alpha_2^6 + \alpha_2^{*6}) + \mathrm{oh}(\lambda t)^3 \tag{27}$$

and

$$<(Z_{1c}(t))^2> = \tfrac{1}{2} + \lambda^2 t^2 (18\,|\,\alpha_2|^2 + 9\,|\,\alpha_2|^4 + |\alpha_2|^6 + 6) + \mathrm{oh}(\lambda t)^3. \tag{28}$$

Therefore

$$[\Delta Z_{1c}(t)]^2 = \tfrac{1}{2} + \lambda^2 t^2 (18\,|\,\alpha_2|^2 + 9\,|\,\alpha_2|^4 + |\alpha_2|^6 + 6) + \mathrm{oh}(\lambda t)^3. \tag{29}$$

since

$$< n_3(t) > = \lambda^2 t^2\ |\alpha_2|^6 + \mathrm{oh}(\lambda t)^3, \tag{30}$$

Thus

$$[\Delta Z_{1c}(t)]^2 - < n_3(t) zx \quad vzxcv + \tfrac{1}{2} > = (18\,|\,\alpha_2|^2 + 9\,|\,\alpha_2|^4 + 6)\lambda^2 t^2 + \mathrm{oh}(\lambda t)^3. \tag{31}$$

which is positive and clearly indicates that there is no squeezing effect in the square of the field amplitude of third harmonic irrespective of the values of t and λ.

From eqs. (25) and (30), the statistics of the third harmonic is seen to be Poissonian as shown by the eqn.

$$< (\Delta n_3(t))^2 > - < n_3(t) > = \mathrm{oh}(\lambda t)^6 \tag{32}$$

5. Conclusion

In this paper, we have shown that suitable values of the interaction time 't' and the coupling parameter 'g' result in the amplitude-squared squeezed states in second harmonic mode during second harmonic generation free from sub-poissonian photon statistics.

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